Journal of
Mechanical
Science and
Technology

Journal of Mechanical Science and Technology 21 (2007) 2244~2249

Viscous dissipation in micro-channels

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(Manuscript Received March 2, 2007; Revised October 24, 2007; Accepted October 29, 2007)

Abstract

Viscous dissipation effects in circular micro-tubes and rectangular micro-channels were investigated theoretically in a fully developed laminar flow region. From the energy conservation law, simple theoretical equations were derived for the evaluation of the viscous dissipation effect. The proposed equations were verified by comparing to other researchers' experimental measurement and numerical results. Wall heat flux, fluid velocity, operating temperature, tube diameter, and working fluids' effects on viscous dissipation were evaluated. It was found that viscous heating increases with decreasing aspect ratio in rectangular channels. The classical macro methods for heat transfer and pressure drop predictions agree well with the measurement and numerical predictions in micro-channels. Finally, a criterion for the evaluation of viscous dissipation was provided with the Brinkman number and heat balance concept.

Keywords: Viscous dissipation; Microfluidics; Laminar flow; Micro-tube; Micro-channel

1. Introduction

With the increased interest in micro-devices, it is highly required to understand the fluid physics in micro tubes. Although much research has been devoted to heat transfer and pressure loss measurements in micro channels, discrepancies among the previous experimental results still exist. Kedzierski [1] reviewed the discrepancy between the macro prediction methods and the previous experimental results in micro-channels and indicated several micro-effects such as velocity slip, temperature jump, viscous dissipation, developing flow, eddy bursting, size dependent viscosity, roughness, compressibility, and flow mal-distribution.

Viscous dissipation represents an increase in internal energy due to the work done on a fluid by viscous forces, that is, a conversion of mechanical energy to thermal energy. Such a process is analogous to frictional work. In many engineering and scientific applications, the dissipation effect is not particularly significant compared with the effect of conduction heat transfer. Viscous dissipation has been regarded as insignificant because of the limit of pumping power. However, viscous dissipation has emerged as an influential factor in microfluidics because tremendous frictional loss can be overcome by high pressure syringe pumps (up to 20,000 psi).

Tunc and Bayazitoglu [2] investigated the viscous dissipation effect with the Brinkman and Knudsen numbers. They carried out numerical analyses for the uniform wall temperature and the constant wall heat flux conditions. Koo and Kleinstreuer [3] also performed a numerical investigation on viscous dissipation effects in micro-tubes and rectangular micro-channels. They introduced water, methanol, and isopropanol as working fluids. They found that viscous dissipation is strongly related to the channel aspect ratio, Reynolds number, Eckert number, Prandtl number and conduit hydraulic diameter. Tso and Mahulikar [4] measured local Nusselt number with Reynolds number along the flow in micro-channels. They

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provided a correlation with the Brinkman number for the flow regime transition and the single-phase forced convective heat transfer in the laminar flow regime in micro-channels for the constant wall heat flux boundary condition. However, their experimental range was the region where viscous dissipation could be neglected. Additionally, their experimental method had a problem with conduction heat transfer through adjacent channels. Their measurement only showed the result of a specific microfluidic device. Xu et al. [5] used the Buckingham Pi theorem to obtain a criterion for the significance of the viscous dissipation effects on micro-tubes. Their own numerical results were used to develop the criterion. They proposed a relatively simple correlation but it produces quite smaller values than the existing measurements and predictions by other researchers. This characteristic will be discussed in the following chapter.

Even though several investigations have been devoted to viscous dissipation, some difficulties still exist in the evaluation of its effect on the flow in micro-tubes. Usually, the proposed criteria are not readily applicable to the design of microfluidic devices because they require solving complex energy equations with the consideration of micro effects like velocity slip and temperature jump. Moreover, the developed correlations use the Brinkman number which can be obtained with the temperature difference between wall and bulk fluid temperatures.

Therefore, a simple prediction method is demanded for the evaluation of the viscous dissipation effect for practical use. In this study, a plain equation was derived from the energy conservation law to evaluate the viscous dissipation effect, and its reliability was verified with the existing measurement and numerically obtained values.

2. Theoretical derivation

Steady state, no work, and no elevation change make the equation of the conservation of energy for open systems a simple form as follows. Especially for incompressible liquids, kinetic energy variation is negligible. Hence, the energy conservation equation is reduced to

$$q''P_e dz = \dot{m}dh \tag{1}$$

For an incompressible substance, the change in enthalpy can be simplified to

$$dh = c_p dT + dP/\rho \tag{2}$$

In micro-channels, the dP/ρ term cannot be neglected because the pressure gradient is significantly large. This term represents the enthalpy change by flow work (frictional heating due to viscous dissipation). The pressure loss can be obtained from the following equation:

$$dP = -f\frac{\rho V^2 dz}{2D_h} \tag{3}$$

Therefore, temperature variation along flow direction can be expressed as

$$\frac{dT}{dz} = \frac{q''P_e}{\dot{m}c_p} - \frac{dP}{\rho c_p} = \frac{q''P_e}{\dot{m}c_p} + \frac{1}{\rho c_p}\left(f\frac{\rho V^2}{2D_h}\right) \tag{4}$$

The pressure loss measurements by Judy et al. [6], Xu et al. [7], Lelea et al. [8] in micro-channels agree well with the conventional macro theory. Therefore, the friction factor can be determined from the conventional macro theory for laminar flow. For circular tubes, the following classic function is being used:

$$f = 64/\mathrm{Re} \tag{5}$$

For rectangular tubes, the following empirical equation with hydraulic diameter (D_h) for macro size tubes is suggested by Rohsenow et al. [9]:

$$f\,\mathrm{Re}_h = 96\left[1 - 1.3553\left(\frac{b}{a}\right) + 1.9467\left(\frac{b}{a}\right)^2 - 1.7012\left(\frac{b}{a}\right)^3 + 0.9564\left(\frac{b}{a}\right)^4 - 0.2537\left(\frac{b}{a}\right)^5\right] \tag{6}$$

$$D_h = 4ab/(a+b) \tag{7}$$

where, a and b is the width and the height of a rectangular channel respectively.

For circular tubes, equation (4) is simplified as follows:

$$\frac{dT}{dz} = \frac{4q''}{c_p\mu\mathrm{Re}} + \frac{32\mathrm{Re}}{c_p D^3}\left(\frac{\mu}{\rho}\right)^2 \tag{8}$$

These are summarized as below:

$\frac{4q''}{c_p \mu \mathrm{Re}}$ temperature rise by conductive heat transfer

$\frac{32\mathrm{Re}}{c_p D^3}\left(\frac{\mu}{\rho}\right)^2$ temperature increase by viscous dissipation

As can be seen in the second term, viscous dissipation is intensified with increasing Re, flow length, and viscosity and decreasing diameter. It is also found that the augmentation of wall heat flux lessens the contribution of viscous dissipation to heat transfer. According to the experimental results of Lelea [8] et al. and Tso and Mahulikar [4], 3-15 kW/m^2 heat fluxes were imposed on 125-700 μm diameter tube walls. Equation (8) shows that the portion of viscous heating in the total heat transfer increases with decreasing wall heat flux. Hence, viscous dissipation plays an important role in small wall heat flux boundary condition. For instance, a tube with 100 micron diameter, 10 cm length, 20°C inlet temperature, the Reynolds number of 500, and a working fluid as water shows 3.6 and 39.7% viscous heating portion in total heat transfer for 50 and 5 kW/m^2 wall heat fluxes, respectively.

Fig. 1 shows the effect of diameter, working fluid, operating fluid temperature, channel length, and Reynolds number (fluid velocity) on viscous dissipation in circular tubes. The values were obtained under adiabatic boundary conditions. As can be expected from equation (8), temperature increases proportionally to the Reynolds number. With increasing working temperature, viscous heating decreases because the physical property of viscosity is reduced with temperature. For a 50 μm diameter tube with water in Fig. 1, the temperature increase by frictional heating reaches almost 2 K at 5 cm. The viscous dissipation effect rises significantly against the reduction of channel diameter. The importance of viscosity can be easily demonstrated by a comparison between refrigerant (low viscosity) and bio-fluids (high viscosity). R134a is the one of the representative refrigerants and its viscous dissipation effect is quite small because its viscosity is much smaller than that of water. The viscosity of R134a at 20°C is only 20% of that of water. On the other hand, a representative bio-fluid, human blood, shows viscous dissipation is an influential factor for fluids with high viscosity even for short flow length. Generally, human blood has three to four times higher viscosity compared to that of water. Some medical diagnostic devices like an enzyme thermistor measure increased temperature by biochemical reaction. Therefore, bio-microfluidic devices like a lab on a chip should consider viscous dissipation effect.

Fig. 1 also shows that Xu et al.'s [5] method (diamond shaped symbols) underestimates the viscous dissipation effect. Usually, their prediction method produces 50% lower value than the values of equation (8) and the numerical results of Koo and Kleinstreuer [3]. Therefore, their method is not appropriate for estimating the effect of viscous dissipation.

Fig. 2 shows the effect of geometry on viscous dissipation. As the aspect ratio deviates from unity, viscous heating increases because of the characteristics of equation (6). This trend also appeared in the work of Koo and Kleinstreuer [3].

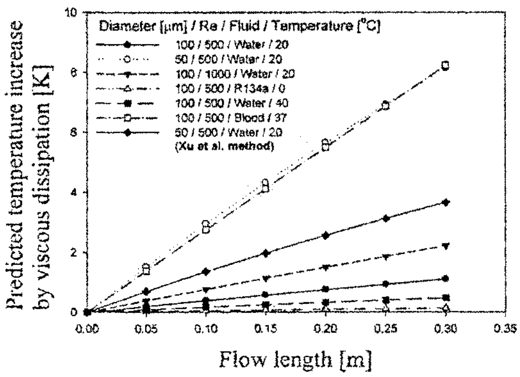

Fig. 1. Temperature increase by viscous dissipation along flow length in adiabatic wall boundary condition.

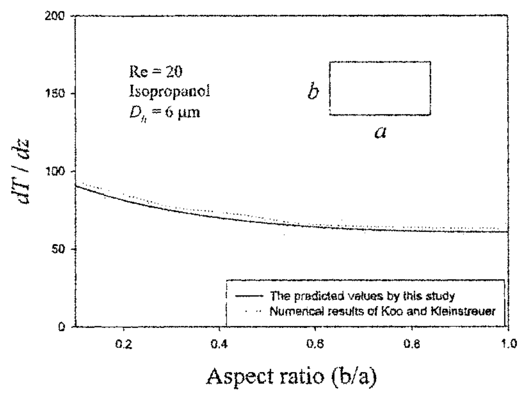

Fig. 2. The effect of geometry on viscous dissipation in rectangular tubes.

3. Comparison with the existing results

Judy et al. [6] measured temperature difference in a 74.1 μm diameter and 11.43 cm length micro-tube with a Reynolds number of 500. The working fluid was isopropanol and an adiabatic wall boundary was applied. The temperature increase by viscous heating was 6.2 K and the predicted value of the equation (8) was 6.487 K. This ensures the reliability of equation (8). The numerical simulation results for rectangular channels by Koo and Kleinstreuer [3] are also compared to the estimated values of the equation (8) in Fig. 3 and also show good agreement.

Koo and Kleinstreuer [3] obtained a temperature gradient along distance for a D_h = 6 μm channel, which corresponds to the 110×3 μm channel case of Pfhaler et al. [10]. They set Re as 20 and the working fluid was isopropanol. Temperature gradients along the aspect ratio were obtained with equations (6-7) and compared with the work of Koo and Kleinstreuer [3]. In the same manner as the circular channel comparison, the proposed equations show reliable performance also.

For heat transfer in macro sized circular ducts, the following equation is suggested by Rohsenow et al. [11]

$$\mathrm{Nu} = \frac{192}{44 + 192\mathrm{Br}} \tag{9}$$

where, Br is defined for the constant wall heat flux boundary.

$$\mathrm{Br} = \frac{\mu V^2}{q'' D_h} \tag{10}$$

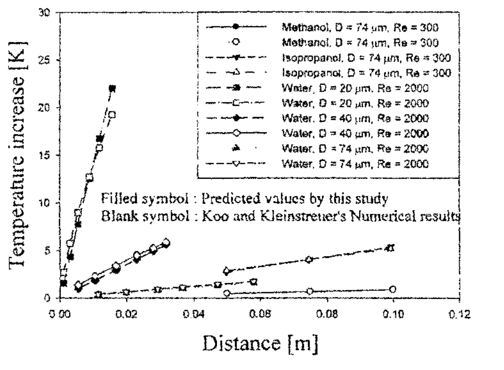

Fig. 3. Comparison between Koo and Kleinstreuer's numerical results and the predicted values by equation (8) for circular micro tubes.

Tunc and Bayazitoglu [2] obtained Nusselt numbers at Br = 0.01 and -0.01. Their numerical results for the corresponding Brinkman numbers are 4.0353 and 4.7481, and equation (9) produces 4.1812 and 4.5627, respectively. The relative differences are within 4%. This shows the classic macro heat transfer prediction method, equation (9), is still effective for the design of microfluidic devices. Even though equation (9) is restricted to continuum flow (Kn<0.001), it is still useful because most liquid flow in microfluidic devices still belongs to the region. The effect of Brinkman number on Nusselt number is shown in Fig. 4.

Xu et al. [7] proposed a non-dimensional criterion for viscous dissipation effect in micro-channels. They drew a line of demarcation with a 3% viscosity variation of water. Unfortunately, their criterion does not consider the effect of heat flux on viscous dissipation. Therefore, a simple criterion for practical usage is proposed to evaluate the effect of viscous dissipation on heat transfer in micro-channels. From equation (4), a relative ratio between conductive heat transfer and frictional heat augmentation by viscous dissipation is simplified to the Brinkman number. It means the Brinkman number is the ratio between frictional heat generation and conductive heat transfer. With the Brinkman number, a criterion is provided with the heat balance concept.

$$\begin{cases} |8\mathrm{Br}| \le |1 - Q_{fluid}/Q_{tube}| \\ \quad \text{: viscous dissipation can be neglected} \\ |8\mathrm{Br}| > |1 - Q_{fluid}/Q_{tube}| \\ \quad \text{: viscous dissipation should be considered} \end{cases} \tag{11}$$

where, Q_{fluid} is the amount of heat transferred to a

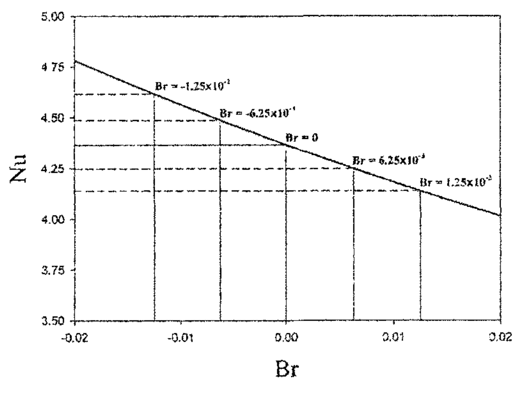

Fig. 4. Nu versus Br.

working fluid and Q_{tube} is the heat addition or removal to a micro-channel.

If the absolute values of $1-Q_{fluid} / Q_{tube}$ are 0.05 and 0.1, the criteria of the Brinkman number are 6.25×10^{-3} and 1.25×10^{-2}, respectively. Even for Lelea et al.'s [8] carefully designed experimental facilities using a vacuum pump for the insulation of a test section to minimize heat loss, $\pm 10\%$ heat balance deviation was reported. Moreover, most of the cases originated from heat loss. It means it is quite difficult to distinguish the effect of viscous dissipation in micro-channels if impeccable thermal insulation is not guaranteed. Experimental uncertainties of heat balance from the previous studies [4, 8] suggest that 3-10% is appropriate for the relative ratio between conductive heat transfer and viscous heating. Lelea et al.'s [8] heat balance results show that 5 and 10% heat loss was detected for 300 and 125.4 μm diameter tubes, respectively. The criterion depends on measurement uncertainties, heat loss, and design parameters. It is recommended to use equation (9) to design microfluidic devices instead of introducing the Nusselt number, 4.364, for the constant heat flux wall condition.

4. Conclusion

Simple equations for the evaluation of viscous dissipation in microchannels were derived from the thermodynamic energy conservation law. The proposed equations were compared to other researchers' measurement and numerical simulation predictions. The proposed method shows a reliable predicting performance and its simplicity has merit over other techniques. It can be used for both circular and rectangular channels. The classic heat transfer prediction method for macro sized tubes is still available for the design of microfluidic devices that should consider viscous dissipation effects in the continuum flow region.

Viscous heating is augmented with increasing fluid velocity, viscosity and decreasing channel diameter, wall heat flux, and operating fluid temperature. It was found that bio- fluids like human blood that have high viscosity should consider viscous dissipation even at low Reynolds numbers. The viscous dissipation effect in rectangular micro-channels was also investigated and it was found that the increase of the aspect ratio augments temperature rise.

Acknowledgments

This work was jointly supported by a Korea University Grant and the Post-doctoral Fellowship Program of Korea Research Foundation (KRF-2004-214-D00020)

Nomenclature

a	:	Channel width [m]
b	:	Channel height [m]
Br	:	The Brinkman number, $\mu V^2 / q''D_h$ [dimensionless]
c_p	:	Specific heat [J/kg K]
D	:	Inner diameter [m]
D_h	:	Hydraulic diameter [m]
f	:	Friction factor [dimensionless]
h	:	Enthalpy [J/kg]
k	:	Thermal conductivity [W/m K]
Kn	:	Knudsen number, l / D [dimensionless]
l	:	Mean free path [m]
$\dot{m}$	:	Mass flow rate [kg/s]
Nu	:	Nusselt number, $\alpha D / k$ [dimensionless]
P	:	Pressure [N/m^2]
P_e	:	Perimeter [m]
q''	:	Heat flux [W/m^2]
Q_{fluid}	:	The amount of heat transferred to a working fluid [W]
Q_{tube}	:	The amount of heat addition or removal to a micro-channel [W]
Re	:	Reynolds number, $\rho VD / \mu$ [dimensionless]
T	:	Temperature [K]
V	:	Mean fluid velocity [m/s]
z	:	Distance along flow direction [m]

Greek symbols

μ	:	Viscosity [kg/m s]
ρ	:	Density [kg/m^3]

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